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A NOTE ON GRAVITATIONAL INSTABILITY

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The purpose of this note is to show that Jeans' criterion for gravitational instability remains unmodified when the viscosity of the medium is assumed to be variable.

Consider an extended viscous and thermally conducting homogeneous medium. The fluctuations in density $\delta\rho$, pressure δp, temperature δT and gravitational potential δv are governed by the following set of equations.

$$\rho \frac{\partial \bar{u}}{\partial t} = -\text{grad}\ \delta p+\rho\ \text{grad}\ \delta v + 1/3\mu\ \text{grad div.}\ \bar{u}+\mu\nabla^2\bar{u}$$

$$+(\text{grad}\ \mu\ .\ \text{grad})\bar{u}+\text{grad}\ \mu\cdot\text{grad}\ \bar{u}-2/3(\text{div}\ \bar{u})\text{grad}\ \mu\ldots$$

$$\frac{d}{dt}\delta\rho = -\rho\ \text{div}\ \bar{u}$$

$$\nabla^2\delta v = -4\pi G\delta\rho \qquad \ldots \quad (1)$$

$$\rho C_p \frac{d}{dt}(\delta T)-\frac{d}{dt}\delta p = K\nabla^2\delta T$$

and

$$\frac{\delta p}{p} = \frac{\delta\rho}{\rho}+\frac{\delta T}{T}$$

We shall seek the solutions which correspond to the propagation of plane waves in the z direction. Then these equations break up into three independent modes.

$$\rho\frac{du_x}{dt} = \mu\frac{d^2u_x}{dz^2}+\frac{du_x}{dz}\cdot\frac{d\mu}{dz} \qquad \ldots \quad (2)$$

$$\rho\frac{du_y}{dt} = \mu\frac{d^2u_y}{dz^2}+\frac{du_y}{dz}\cdot\frac{d\mu}{dz} \qquad \ldots \quad (3)$$

and

$$\rho\frac{du_z}{dt} = -\frac{d}{dz}\delta p+\rho\frac{d}{dz}\delta v+\tfrac{4}{3}\mu\frac{d^2u_z}{dz^2}+\tfrac{4}{3}\frac{du_z}{dz}\cdot\frac{d\mu}{dz}\ ;$$

$$\frac{d}{dt}\delta\rho = -\rho\frac{du_z}{dz}\ ;\quad \frac{d^2}{dz^2}(\delta v) = -4\pi G\delta\rho \qquad \ldots \quad (4)$$

$$\frac{d}{dt}(\delta p-c^2\delta\rho) = \frac{K}{\rho c_v}\frac{d^2}{dz^2}(\delta p-c'^2\delta\rho)$$

where c, c' denote the adiabatic and isothermal velocity of sound and K the thermal conductivity. Two modes of wave propagation given by equations (2) and (3) are trival and unaffected by graviy, compressibility and conductivity as expected. For a solution of equation (4) we write

$$\frac{d}{dt} = i\omega \quad \text{and} \quad \frac{d}{dz} = -ik \tag{5}$$

where ω denotes the frequency and k the wave number. Substitute equation (5) in equation (4), we obtain a system of linear homogeneous equations which can be written in the matrix notation in the form

$$\left|\begin{matrix} i\rho\omega + \frac{8}{3}\mu k^2 & i\rho k & -ik & 0 \\ -i\rho k & 0 & 0 & i\omega \\ 0 & -k^2 & 0 & 4\pi G \\ 0 & 0 & \left(i\omega + \frac{K}{\rho c_v} k^2\right) & -\left(i\omega c^2 + \frac{K}{\rho c_u} c'^2\right) \end{matrix}\right| \left|\begin{matrix} u_z \\ \delta v \\ \delta p \\ \delta\rho \end{matrix}\right| = 0 \quad \ldots \tag{6}$$

This is identical with that obtained by Kato and Kumar (1960) except that the coefficient of μk^2 in the first term of the first column is 8/3, instead of 4/3. Hence it can be concluded that there will be no modification of Jeans' Criterion for gravitational instability, when the viscosity of the medium is assumed to be variable.

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